Superconformal Blocks and Calogero-Moser Systems

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DESY

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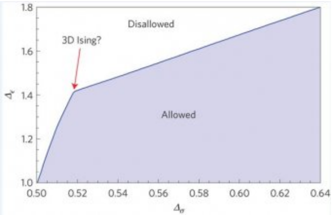
based on I.B, V. Schomerus and E. Sobko, [1904.04852]

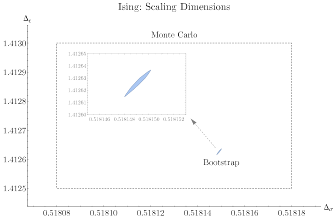
Conformal field theory

Conformal symmetry $\rightarrow$ self-similar systems
CFT-s appear in statistical physics, holography, string theory...

2d theories, $\mathcal{N} = 4$ SYM, fishnets studied by a variety of methods, including integrability

Bootstrap: derive general properties of CFT-s from consistency



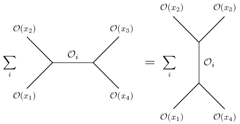


Conformal bootstrap

Mathematically, CFT = unitary rep $\mathcal{H}$ of the conformal group $G = SO(d+1,1)$ + OPE.
Correlation functions are expanded in *conformal blocks*

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_4(x_4)\rangle = \sum_i \sum_\psi \langle \mathcal{O}_1\mathcal{O}_2|\psi\rangle\langle\psi|\mathcal{O}_3\mathcal{O}_4\rangle \sim \sum_i g_i.$$

Associativity of OPE implies *crossing symmetry*



Aim of this talk: mathematics of conformal blocks

Harmonic analysis

A simple map between scalar conformal blocks and BC_2 Calogero-Moser-Sutherland wavefunctions [Isachenkov, Schomerus]

$$\psi(x_1, x_2) = \prod_{i=1}^{2} \frac{(z_i - 1)^{\frac{a+b}{2}+\frac{1}{4}}}{z_i^{\frac{1}{2}+\frac{\epsilon}{2}}} g(z_1, z_2), \;\; z_i = -\sinh^{-2}\frac{x_i}{2}.$$

The origin of this relation lies in harmonic analysis on G [Isachenkov, Schomerus, Sobko]

$$g \in (\pi_1 \otimes \pi_2 \otimes \pi_3 \otimes \pi_4)^G.$$

Blocks are sections of a vector bundle over the double coset

$$\mathcal{M} = K\backslash G/K, \;\; K = SO(1,1) \times SO(d).$$

and are eigenfunctions of the Laplace-Beltrami operator.

Applications to field theory

Idea: use known integrable structure of Calogero-Moser models to derive interesting properties of conformal blocks

Applications include: defect blocks $\to BC_N$, inversion formulae, weight shifting operators...

Supersymmetry? Work with *algebraic principal series*

$$\pi_i = \mathrm{Hom}_{U(\mathfrak{p})}(U(\mathfrak{g}), V_i)$$

For type I Lie superalgebras, the Laplacian receives a nilpotent correction

$$\Delta = \Delta_0 - 2D^{ab}\partial_a\partial_b.$$

which allows for a solution by *finite* perturbation theory.

Thank you!